Unifying Treatment to Control of Nonlinear Systems with Two Timescales

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I. Introduction

BECAUSE of the difficulty associated with the analysis of general nonlinear dynamic systems, the challenge of control law design for nonlinear systems is often met by exploiting special features of a class of systems. Examples include the systems that are feedback/input-output linearizable,[1] differentially flat,[2] or possess certain unique structure that enables the application of Lyapunov-function-based methods such as backstepping.[3] Nonlinear systems with multiple timescales are another case where singular-perturbation-based approaches may be developed for control design. In aerospace engineering applications it has long been recognized that multiple timescales are present in many aerospace problems: the attitude kinematics are generally faster than the position kinematics; the angular rate dynamics in turn are faster than the attitude kinematics. This timescale separation has been employed in trajectory optimization and analysis[4–7] and flight-control problems[8–11] to reduce the complexity. The commonality in these two types of problems is that some of the fast state variables are treated as quasi-controls in the reduced-order (outer) solution. A major difference between them lies in that the solution in the boundary layer (the inner solution) usually needs to be analyzed and obtained for boundary condition matching in trajectory optimization, whereas stability is the main concern for flight-control applications. Therefore an outer solution preserving stability is necessary in control applications. This Note focuses on the control design aspect. In particular, we attempt to unify the approaches using timescale separation in various previous work in flight control[8–11] under one framework, extend the formulation of the type of problems to a more general setting, and formalize the procedure and the associated stability analysis. Ultimately, the hope is that with this development a systematic synthesis procedure for control design can be followed in other applications where timescale separation exists.

In Sec. II a general formulation of the problems is presented, a procedure for control law design is proposed, the conditions that must be satisfied are stated, and the stability of the closed-loop system is analyzed. An illustrative application is provided in Sec. III, where the entry trajectory control problem for a suborbital prototype of reusable launch vehicle (X-33) is used to demonstrate the methodology. Conclusions are given in Sec. IV.

II. Methodology

In this section the model of the type of systems under consideration is described. The procedure and conditions for constructing the control law is presented. The closed-loop stability is discussed.

A. Formulation

We consider a nonlinear system for which the dynamics have two timescales and can be expressed as

$$\dot{x}_1 = f_1(t, x_1, x_2, x_3, u_1) \tag{1}$$

$$\varepsilon\dot{x}_2 = f_2(t, x_1, x_2, x_3, u_1, u_2), \qquad 0 < \varepsilon \ll 1 \tag{2}$$

$$\dot{x}_3 = f_3(t, x_1, x_2, x_3, u_1, u_2) \tag{3}$$

where $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$, $x_3 \in R^{n_3}$, and $x = \mathrm{col}(x_1\ \ x_2\ \ x_3) \in R^n$ with $n = n_1 + n_2 + n_3$ is the state vector; $u_1 \in R^{m_1}$, $u_2 \in R^{m_2}$, and $u = \mathrm{col}(u_1\ \ u_2) \in R^m$ with $m = m_1 + m_2$ is the control vector. Note that f_1 is independent of u_2. We will require

$$n_2 + m_1 = n_1 \tag{4}$$

$$m_2 \ge n_2 \tag{5}$$

The condition (4) is not always necessary (see Remark 2). Each part of the state and control is restricted to a connected set: $x_i \in X_i$, $i = 1, 2, 3$, and $u_j \in U_j$, $j = 1, 2$. We will denote $X = \cup X_i$ and $U = \cup U_j$. The right-hand sides of the system equations (1–3) are assumed to be sufficiently smooth to satisfy the usual conditions for existence of solutions and support the continuity requirements. The control objective is to find a control law for u such that $x_1(t)$ tracks a given smooth and bounded function $x_1^*(t) \in X_1$, for all $t \ge 0$.

Remarks:

1) The modeling of the system (1–3) encompasses the dynamics of the systems in previous studies as special cases. For instance, in Refs. 9 and 10 the airplane attitude kinematics would be the slow subsystem f_1 (and f_1 is also independent of u_1 in that case). The angular rate dynamics would be the fast subsystem f_2. The rest of the airplane dynamics of position, altitude, and speed, although not considered expressively in Refs. 9 and 10, would be in f_3.

2) The condition $n_2 + m_1 = n_1$ is not a must. When $x_1^*(t)$ is an arbitrary function, the problem is an output-tracking problem. In this case usually it will be required that $n_2 + m_1 = n_1$ because as will be seen later that $v = (x_2\ \ u_1)$ is used as the pseudocontrol for subsystem (1). In such a case the following condition will also be needed:

$$\frac{\partial f_1(t, x_1, x_2, x_3, u_1)}{\partial v} \quad \text{is full rank}$$
$$\forall x \in X, \quad u_1 \in U_1, \quad t \ge 0 \tag{6}$$

where the ith row of the Jacobian $\partial f_1/\partial(x_2\ \ u_1)$ is defined to be

$$\left(\frac{\partial f_{1i}}{\partial v_1} \cdots \frac{\partial f_{1i}}{\partial v_{n_2}} \cdots \frac{\partial f_{1i}}{\partial v_{n_2+m_1}}\right)$$

Exceptions to the requirement of $n_2 + m_1 = n_1$ include the cases where part of the subsystem (1) is related to the other part by pure integrators. And when $x_1^*(t)$ is from a reference trajectory of the system (1–3), that is, a given pair $[x^*(t), u^*(t)] \in X \cup U$ that satisfies (1–3), the problem is a regulation problem, and the condition $n_2 + m_1 = n_1$ is not necessary. The strategy described in the following will be applicable in more general cases. We stipulate this condition of $n_2 + m_1 = n_1$ nonetheless to avoid to have to repeat the same comments many times.

3) To ensure the possibility of second step in control law synthesis described in the next section, the following condition similar to Eq. (6) is required:

$$\frac{\partial f_2(t, x_1, x_2, x_3, u_1, u_2)}{\partial u_2} \quad \text{is full rank}$$
$$\forall x \in X, \quad u \in U, \quad t \ge 0 \tag{7}$$

4) When $X \cup U$ contains the origin and $x_1^*(t) \equiv 0$, the problem reduces to a stabilization problem. If the objective is to stabilize the complete state vector x at the origin, further condition on the stability of the zero dynamics of the system will be needed to ensure $x(t) \to 0$ as $x_1(t) \to 0$, that is, the system must be a minimum-phase system with x_1 as the output.[1]

B. Control Synthesis

The idea is to use the fast variable x_2 as pseudocontrol in the first step of the control law design process. If the slow dynamics [Eq. (1)] can be controlled to track $x_1^*(t)$ with x_2 and u_1 as inputs, a control

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law for u_2 will be sought to drive the fast dynamics [Eq. (2)] toward the x_2 value required to control to the slow dynamics.

We shall restrict our discussion to static feedback control laws. Suppose that continuous feedback laws

$$u_1 = v_1(t, x_1, x_3) \tag{8}$$

$$x_2 = w_2(t, x_1, x_3) \tag{9}$$

have been found, and the resulting slow dynamics become

$$\dot{x}_1 = f_1(t, x_1, w_2, x_3, v_1) \stackrel{\Delta}{=} h_1(t, x_1, x_3) \tag{10}$$

Let $e_1 = x_1 - x_1^*$, and $\dot{e}_1 = h_1(t, x_1, x_3) - \dot{x}_1^*(t) = \eta(t, e_1, x_3)$, when the substitution $x_1 = e_1 + x_1^*(t)$ is made. The condition on the construction of Eqs. (8) and (9) is as follows.

Requirement 1:

The feedback control law (8) and the pseudocontrol law (9) can be found such that 1) $v_1 \in U_1$ and $w_2 \in X_2, \forall x_1 \in X_1, x_3 \in X_3$ and $t \geq 0$. 2) The error dynamics $\dot{e}_1 = \eta(t, e_1, x_3)$ are uniformly asymptotically stable at $e_1 = 0$ for any $x_3 \in X_3$.

Remarks:

1) When f_1 is affine in x_2 and u_1, it is straightforward to construct the control laws (8) and (9), for instance, by the input-output feedback linearization technique.[1] When f_1 is not affine in x_2 and u_1, other methods exist that might be workable.[12,13] Because only the subsystem f_1 is being considered, it is more likely to find an existing nonlinear control method applicable to it than to the overall system.

2) A special case is when $m_1 = 0$ as in aircraft flight-control applications in Refs. 9–11. In such a case x_2 will be treated as the sole input to control subsystem (1).

To finish the control law synthesis, define $\delta = x_2 - w_2(t, x_1, x_3)$. The next step is to find a control law for u_2:

$$u_2 = v_2(t, x_1, x_2, x_3, v_1, \delta) \tag{11}$$

where v_2 is continuous. Because v_1 and w_2 are functions of x_1, x_3, and t, and v_2 a function of x_1, x_2, x_3, and t, we can denote, with slight abuse of notation, $u_2 = v_2(t, x_1, x_2, x_3)$; therefore, $f_2(t, x_1, x_2, x_3, v_1, v_2) \stackrel{\Delta}{=} h_2(t, x_1, x_2, x_3)$. Rewrite the system equation (2) under control laws v_1 and v_2 as

$$\dot{x}_2 = h_2(t, x_1, x_2, x_3) \tag{12}$$

and let the Jacobian $J(t, x_1, x_2, x_3) = \partial h_2/\partial x_2$. The conditions we need v_2 to satisfy are then the following.

Requirement 2:

1) $v_2 \in U_2$ for all $x \in X$ and $v_1 \in U_1$.

2) Every solution of the algebraic equation $h_2(t, x_1, x_2, x_3) = 0$ necessitates $x_2 = w_2$.

3) All of the eigenvalues of the Jacobian $\partial h_2/\partial x_2 = J(t, x_1, x_2, x_3)$ have negative real part for any fixed $x \in X$ and $t \geq 0$.

The assumption that $n_2 = m_2$ in the subsystem (2) makes it possible for the preceding conditions, especially 2 and 3, to be realizable. For instance, when f_2 is affine in u_2,

$$f_2 = \beta(t, x) + g_2(t, x)u_2$$

then $u_2 = g_2^{-1}[-\beta + P(x_2 - w_2)]$ with any $n_2 \times n_2$ Hurwitz matrix P satisfies conditions 2 and 3. This is the case in flight-control applications.[9–11] But it is certainly far from the only possibility. For nonaffine cases different methods can be exploited for construction of the control law (11).[12,13]

C. Stability Analysis

To establish the stability/boundedness of the state and control for the complete system, we also need the following to be true.

Requirement 3:

Let $h_3(t, x_3)$ be the right-hand side of subsystem (3) when $x_1 \equiv x_1^*(t)$, $x_2 \equiv w_2[t, x_1^*(t), x_3]$, $u_1 = v_1[t, x_1^*(t), x_3]$, and $u_2 = v_2\{t, x_1^*(t), w_2[t, x_1^*(t), x_3], x_3\}$ for all $x_3 \in X_3$. All of the solutions of the system in x_3,

$$\dot{x}_3 = h_3(t, x_3) \tag{13}$$

are bounded.

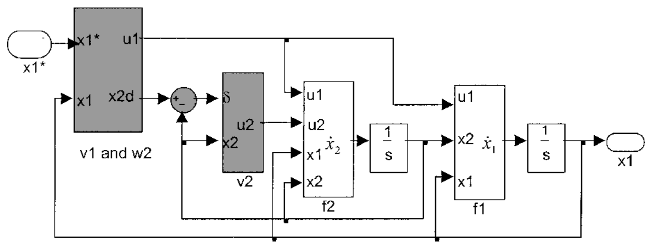

Fig. 1 Block diagram of the proposed control strategy.

With requirements 1–3 met we can conclude the following:

The response of the system (1–3) is such that $x_1(t) \to x_1^*(t)$, $x_2(t)$, $x_3(t)$, $u_1(t)$, and $u_2(t)$ are all bounded, when ε is sufficiently small.

This conclusion can be deduced by using the standard singular perturbation theory and the continuity of the functions involved. Under a control law (11) that satisfies condition 3 in requirement 2, the fast dynamics (2) has an attracting asymptotic solution (outer solution) to which the solutions of the subsystem (2) will quickly converge.[14] The zeroth-order outer solution is given by $f_2(t, x_1, x_2, x_3, v_1, v_2) = 0$, which, by condition 2 in requirement 2, necessitates $x_2 \to w_2(t, x_1, x_3)$. By requirement 1, $u_1 = v_1(t, x_1, x_3)$ and $x_2 = w_2(t, x_1, x_3)$ will result in $x_1(t) \to x_1^*(t)$. All of the preceding will produce a bounded $x_3(t)$ (requirement 3). The boundedness of $x_3(t)$ and $x_1(t)$ ensures that of $v_1(t, x_1, x_3)$ and $w_2(t, x_1, x_3)$; thus the boundedness of $u_1(t)$ and $x_2(t)$. Finally, the boundedness of the x leads to the boundedness of $u_2(t)$. All of the analysis presented here is accurate to the order of ε.

Remarks:

1) Although requirement 3 is not explicitly discussed in previous flight-control applications,[9–11] it is satisfied by the aircraft dynamics because the subsystem (3) would represent the altitude and velocity dynamics. This requirement is necessary for the approach to be applicable to other nonlinear systems.

2) For stabilization problems ($x_1^* = 0$ in such a case), if further conditions are imposed:

$$v_1(t, 0, 0) = 0, \qquad w_2(t, 0, 0) = 0$$

$$v_2(t, 0, 0, 0) = 0, \qquad \forall t \geq 0$$

and if Eq. (13) is uniformly asymptotically stable at $x_3 = 0$, then the overall system is uniformly asymptotically stable at the origin. Similar stability analysis and results are well documented in the literature.[15]

Figure 1 is a block diagram that illustrates the control design approach. The two gray blocks are the control laws [Eqs. (8), (9), and (11)], with $x_{2d} = w_2$. The dependence of all the blocks on x_3 has been supressed in the figure for clarity. The basic concept can be summarized as follows: the control law for u_2 causes $\delta = x_2 - x_{2d} \to 0$ rapidly; then the control law for u_1 and $x_2 = w_2$ drive $x_1(t) \to x_1^*(t)$ asymptotically, while all of the signals remain bounded.

III. Application in Entry Trajectory Control

A. Guidance Development

Entry guidance is concerned with providing bank-angle and angle-of-attack commands for a spacecraft or reusable launch vehicle when it reenters the atmosphere. The prevalent entry guidance methods employ a reference trajectory obtained off-line and generate onboard feedback commands to guide the vehicle to track some key parameters defined by the reference. The space shuttle entry guidance tracks the reference drag-acceleration profile for downrange distance control[16]; following, the reference ground-track with approximate feedback linearization guidance law is investigated[17]; tracking the complete reference trajectory in the state space is also a possibility.[18] We shall use the entry guidance problem to demonstrate that the methodology discussed in this Note leads to another very simple strategy for following the reference ground track.

Define the specific energy of the vehicle by

$$e = \mu/r - V^2/2 \tag{14}$$

Then it can be easily shown that $de/dt = DV/m > 0$ so that e is a monotonically increasing variable, where D is the aerodynamic drag force, V the Earth-relative velocity, and m the mass of the vehicle. Using e as the independent variable, the three-degree-of-freedom equations of motion over a nonrotating spherical Earth are given by

$$\dot{\theta} = \frac{\cos\gamma}{r\cos\phi}\left(\frac{\sin\psi}{D/m}\right) \tag{15}$$

$$\dot{\phi} = \frac{\cos\gamma}{r}\left(\frac{\cos\psi}{D/m}\right) \tag{16}$$

$$\dot{\psi} = \frac{m}{DV^2}\left(\frac{L\sin\sigma}{m\cos\gamma} + \frac{V^2}{r}\cos\gamma\sin\psi\tan\phi\right) \tag{17}$$

$$\dot{r} = \frac{\sin\gamma}{D/m} \tag{18}$$

$$\dot{\gamma} = \frac{m}{DV^2}\left[\frac{L}{m}\cos\sigma + \left(V^2 - \frac{\mu}{r}\right)\frac{\cos\gamma}{r}\right] \tag{19}$$

where μ is the gravitational parameter; r the radial distance from the center of the Earth to the entry vehicle; the longitude and latitude are θ and ϕ, respectively; the Earth-relative velocity is V; L and D are aerodynamic lift and drag forces, which are functions of r, V, and angle of attack α; γ is the flight-path angle; σ the bank angle; finally, the velocity azimuth angle is ψ, measured from the North in a clockwise direction. The derivatives are with respect to e, and with e as the independent variable V is not an independent state because $V = \sqrt{[2(\mu/r - e)]}$. Therefore no equation for V is necessary in the equations of motion. The preceding system is nonlinear and time-varying. Suppose that a reference trajectory based on system (15–19) has been obtained, denoted by $[\theta^*(e), \phi^*(e), \psi^*(e), r^*(e), \gamma^*(e)]$ and $[\alpha^*(e), \sigma^*(e)]$. The initial and final energy e_0 and e_f are fixed. At any given energy level $e_0 \le e \le e_f$ the altitude and velocity are related by Eq. (14) and are usually not far from their nominal values. If the emphasis of the entry guidance is on accurate ground-track following (such as the case where the vehicle is desired not to fly over a specified location on the ground), a conceptually simple guidance law can be developed using the methodology discussed in Sec. II.

The reference ground-track is given by $[\theta^*(e), \phi^*(e)]$. It has long been recognized that the position dynamics equations (15) and (16) are slower that the attitude dynamics [Eq. (17)]. Therefore according to the formulation in Sec. II.A, we can partition the entry vehicle dynamics by $x_1 = (\theta, \phi)$, $x_2 = \psi$, and $x_3 = (r, \gamma)$. The trajectory controls are $u_1 = \alpha$ and $u_2 = \sigma$. The conditions mentioned in Remarks 1–3 of Sec. II.A are all met if $-90\ \deg \le \sigma \le 90\ \deg$ is assumed. Following the strategy in Sec. II.B, we will first find a pseudocontrol law for ψ and a control law for α such that $\theta \to \theta^*$ and $\phi \to \phi^*$. To this end, denote $v_1 = m\sin\psi/D$ and $v_2 = m\cos\psi/D$. Choose v_1 and v_2 to be

$$v_1 = \frac{r\cos\phi}{\cos\gamma}\left[-\dot{\theta}^* - k_1(\theta - \theta^*)\right] \tag{20}$$

$$v_2 = \frac{r}{\cos\gamma}\left[-\dot{\phi}^* - k_2(\phi - \phi^*)\right] \tag{21}$$

for some $k_1 > 0$ and $k_2 > 0$. These choices of v_1 and v_2 will yield the position dynamics to be

$$\Delta\dot{\theta} = -k_1\Delta\theta \tag{22}$$

$$\Delta\dot{\phi} = -k_2\Delta\phi \tag{23}$$

where $\Delta\theta = \theta - \theta^*$ and $\Delta\phi = \phi - \phi^*$; thus, $\theta \to \theta^*$ and $\phi \to \phi^*$. The pseudocontrol law for ψ is then from v_1 and v_2 by

$$\tilde{\psi} = \tan^{-1}(v_1/v_2) \tag{24}$$

The required α is numerically solved at each r and V to satisfy

$$D(\alpha) = m\Big/\sqrt{v_1^2 + v_2^2} \tag{25}$$

The Newton iteration is generally effective for this task. In the absence of saturation, clearly condition 2 of requirement 1 in Sec. II.B is satisfied by $\tilde{\psi}$ and α so obtained.

The next step is to design the control law for σ such that the three conditions of requirement 2 in Sec. II.B are met. Based on Eq. (17), the following choice of σ will meet conditions 2 and 3 in requirement 2:

$$\sin\sigma = (m\cos\gamma/L)\left[-(V^2/r)\cos\gamma\sin\psi\tan\phi - k_3(\psi - \tilde{\psi})\right] \tag{26}$$

where $k_3 > 0$. With this choice of σ, Eq. (17) becomes

$$\dot{\psi} = -\left(k_3 m\big/DV^2\right)(\psi - \tilde{\psi}) \triangleq f_\psi(r, V, \psi, \tilde{\psi}) \tag{27}$$

Now clearly the unique solution for setting the right-hand side of the fast dynamics (27) $f_\psi = 0$ is $\psi = \tilde{\psi}$ (condition 2 of requirement 2). And at any given state $D > 0$ and $V > 0$; hence, $\partial f_\psi/\partial\psi = -k_2 m/DV^2 < 0$ (condition 3 of requirement 2). By the analysis in Sec. II.C under the guidance laws (25) and (26), the reference values of θ^* and ϕ^* will be tracked asymptotically in the absence of saturation.

Finally, requirement 3 in Sec. II.C needs to be examined. Although it is not easy to verify requirement 3 analytically, understanding of the physics of the problem, as in many other cases, will render an indirect verification. In this case r and V will not depart significantly from their nominal values because the binding of the independent variable energy e, provided that no very large initial dispersions in r and V take place. Therefore when the reference ground-track is followed precisely, $x_3 = (r, \gamma)$ can only be bounded (and close to their nominal values). In fact, based on the dynamic equations (15–19), we can further conclude that ψ and the bank angle σ will approach their respective nominal values as the reference ground-track (θ^*, ϕ^*) is tracked.

B. Numerical Illustrations with the X-33 Data

The X-33 was a half-scale prototype for reusable launch vehicle (RLV) designed to test key technologies for full-scale RLVs. In flight testing the X-33 was to be launched vertically from Edwards Air Force Base, accelerate to about Mach 9 at an altitude of 55 km, and land horizontally at the Michael Army Air Field in Utah. The entry guidance system is responsible for guiding the X-33 shortly after the main-engine cutoff until the X-33 reaches the terminal area energy management (TAEM) interface at about 56 km (30 nm) away from the heading alignment cone near the end of the runway. The entry trajectory is controlled by bank-angle and angle-of-attack modulations.

A reference entry trajectory is designed using the X-33 data and trajectory constraints. Suppose that the reference ground track is to be tracked. The guidance laws for α and σ are derived as in Sec. III.A. To test the effectiveness of the approach, random initial conditions dispersions are added to the simulations. Figure 2 shows the tracking errors along a number of such trajectories in downrange and cross-range directions, defined by $R_0\Delta\theta$ and $R_0\Delta\phi$, where $R_0 = 6378$ km is the radius of a spherical Earth. The convergence onto the reference ground-track is evident, and both the downrange and cross-range errors at the TAEM interface were all less than 1 km. Figure 3 depicts the dispersions of the rest of the state variables. As expected, $\Delta\psi$ approached zero, and the dispersions of r, V, and γ were all bounded and acceptable. The variations of σ and α are in Fig. 4. The bank angle σ by and large converged to its nominal profile.

The main purpose here is to demonstrate the control synthesis procedure established in Sec. II, not necessarily to propose a new entry guidance approach. Much more extensive testing under various entry constraints and requirements would be needed for that purpose. In fact, the trajectory regulation approach in Ref. 18 gives significantly higher overall guidance precision than seen in Fig. 3. On the other hand, the approach based on timescale separation design is much simpler and provides good performance on what it was intended to achieve (ground-track following). Another added

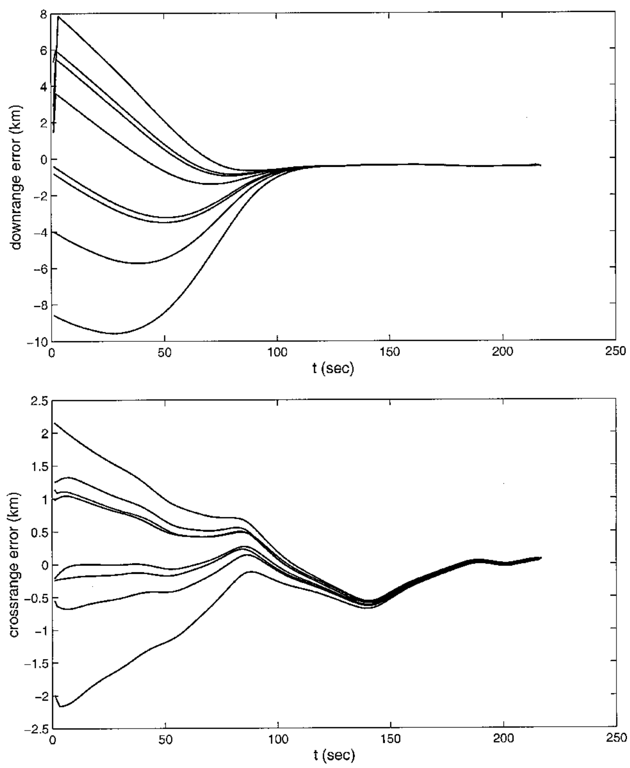

Fig. 2 Tracking errors in ground track.

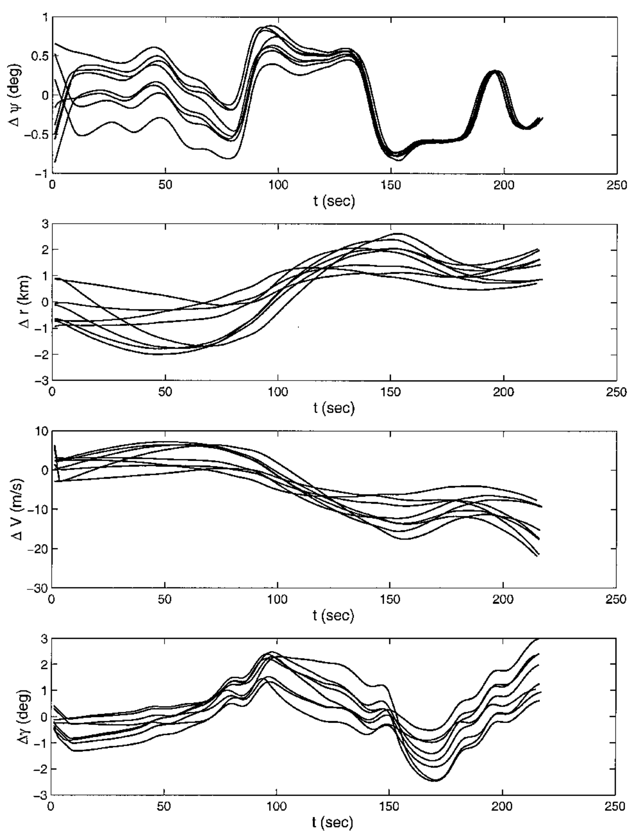

Fig. 3 Variations of dispersion in other state variables.

benefit of the current approach is that the convergence of ψ to its outer solution $\tilde{\psi}$ will be very robust with respect to modeling uncertainty in the lift-to-drag ratio (L/D). To see this, assume that a coordinate transformation has been used to rotate the plane so that the equatorial plane contains the initial and target points of the entry flight. In such a case $\phi \approx 0$ along the entry trajectories, and Eq. (17) reduces to

$$\dot{\psi} = (1/V^2 \cos\gamma)(L/D)\sin\sigma \tag{28}$$

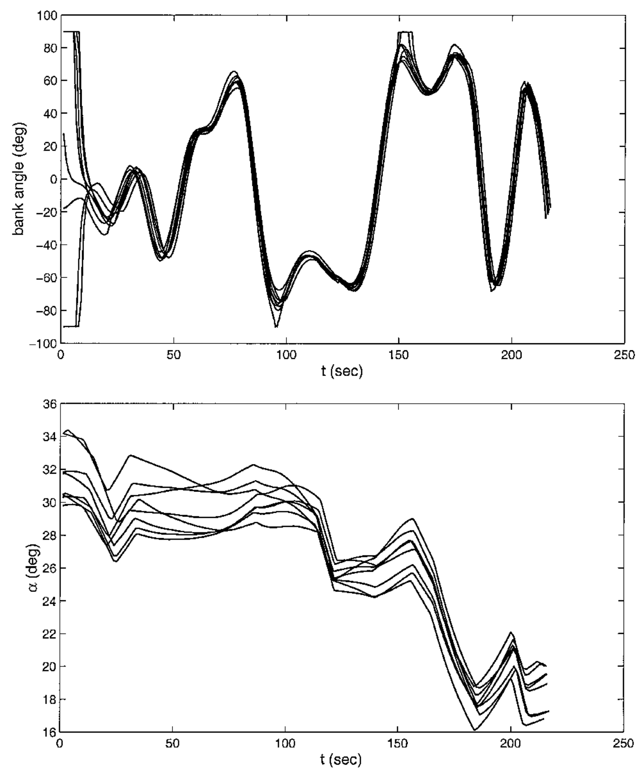

Fig. 4 Bank-angle and angle-of-attack variations.

The corresponding bank-angle control law in this case can be chosen to be independent of aerodynamic forces

$$\sin\sigma = -k_3(\psi - \tilde{\psi}) \tag{29}$$

From Eqs. (28) and (29) we have

$$\dot{\psi} = -(k_3/V^2 \cos\gamma)(L/D)(\psi - \tilde{\psi}) \tag{30}$$

Clearly conditions 2 and 3 of requirement 2 in Sec. II.B are satisfied; thus, $\psi \to \tilde{\psi}$, regardless of the value of L/D.

IV. Conclusions

A general formulation of the control problem for nonlinear systems with two timescales is presented. This formulation unifies various aerospace applications in previous work under one framework and extends further beyond. Control synthesis procedures exploiting timescale separation are given, and the necessary requirements that should be met in the process are established. The associated stability analysis is formalized. A problem of entry trajectory control is solved to provide a step-by-step demonstration of the procedure and numerical illustrations. What remains to be investigated is the general issue of robustness of this approach with respect to external disturbances and modeling errors.

Acknowledgments

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Nonlinear Tracking Control of an Underactuated Spacecraft

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Introduction

RECENTLY, there has been some interest in designing controllers for the underactuated rigid spacecraft tracking/stabilization problem. In Ref. 1, Crouch provided necessary and sufficient conditions for controllability of a rigid body in the case of one, two, or three independent actuators. In Ref. 2, Byrnes and Isidori demonstrated that a rigid spacecraft with only two controls cannot be asymptotically stabilized via continuous-state feedback because it does not satisfy Brockett's necessary condition[3] for smooth feedback stabilizability. In Ref. 4, Morin et al. developed a smooth, time-varying stabilizing controller by using averaging theory. A continuous time-varying/time-periodic switching controller was proposed by Coron and Kerai in Ref. 5. When averaging theory and Lyapunov control design techniques were used, Morin and Samson[6] developed a continuous time-varying controller that locally exponentially stabilized the attitude of a rigid spacecraft. Recently, in Ref. 7, Tsiotras and Luo proposed a saturated, tracking/stabilizing controller for the kinematic control of an underactuated axisymmetric spacecraft; however, the spin rate on the unactuated axis is required to be zero.

In this Note, we propose a novel, continuous, time-varying, nonlinear tracking controller for the kinematic model of an axisymmetric as well as nonaxisymmetric underactuated rigid spacecraft. The control structure is motivated by the Lyapunov-based dynamic oscillator presented in Ref. 8 for wheeled mobile robots. The proposed control approach is novel in that several key characteristics of the quaternion kinematic representation are exploited during the redesign of the control structure originally proposed in Ref. 8. Indeed, it is the fusion of the dynamic oscillator-based approach, that is, it provides additional design flexibility, and quaternion kinematic representation that facilitates the tracking result for the underactuated spacecraft system. The controller ensures that the spacecraft orientation error is driven to an arbitrarily small neighborhood of zero provided the initial errors are sufficiently small, that is, the controller guarantees local uniform, ultimately bounded (LUUB) tracking with exponential convergence. Standard backstepping control techniques can be fused with the kinematic controller to present a solution, that is, both dynamic and kinematic effects can be accounted for, for full-order LUUB tracking/regulation of an axisymmetric spacecraft. In contrast to the work presented in Ref. 7, the axisymmetric control strategy is for the full-order model; furthermore, it does not impose restrictions on the spin rate of the unactuated axis.

Problem Formulation

The kinematics for a rigid spacecraft can be expressed as follows[9]:

$$\dot{\boldsymbol{q}} = \tfrac{1}{2}(q^{\times}\omega + q_0\boldsymbol{\omega}) \tag{1}$$

$$\dot{q}_0 = -\tfrac{1}{2}\boldsymbol{q}^T\boldsymbol{\omega} \tag{2}$$

where $\boldsymbol{\omega}(t) \in \Re^3$ is the angular velocity of a body-fixed reference frame $\mathcal{F}$ (located at the center of mass of the spacecraft) with respect to an inertial reference frame $\mathcal{I}$, the notation $\zeta^{\times}$, $\forall \zeta = [\zeta_1 \;\; \zeta_2 \;\; \zeta_3]^T$, denotes the following skew-symmetric matrix:

$$\zeta^{\times} = \begin{bmatrix} 0 & -\zeta_3 & \zeta_2 \\ \zeta_3 & 0 & -\zeta_1 \\ -\zeta_2 & \zeta_1 & 0 \end{bmatrix} \tag{3}$$

and $\boldsymbol{q}_E(t) \triangleq \{q_0(t), \boldsymbol{q}(t)\} \in \Re \times \Re^3$ represents the unit quaternion[9] describing the orientation of the body-fixed frame $\mathcal{F}$ with respect to the inertial frame $\mathcal{I}$, which is subject to the constraint $\boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1$. The rotation matrix that brings $\mathcal{I}$ onto $\mathcal{F}$, denoted by $R(\boldsymbol{q}, q_0) \in \Re^{3\times3}$, is defined as

$$R \triangleq \left(q_0^2 - \boldsymbol{q}^T\boldsymbol{q}\right)I_3 + 2\boldsymbol{q}\boldsymbol{q}^T - 2q_0\boldsymbol{q}^{\times} \tag{4}$$

where I_3 is the 3×3 identity matrix and the angular velocity of $\mathcal{F}$ with respect to $\mathcal{I}$ expressed in $\mathcal{F}$ is $\boldsymbol{\omega}(t)$.

We assume that the desired attitude of the spacecraft can be described by a desired, body-fixed reference frame $\mathcal{F}_d$ whose orientation with respect to the inertial frame $\mathcal{I}$ is specified by the desired unit quaternion $\boldsymbol{q}_{dE}(t) = \{q_{0d}(t), \boldsymbol{q}_d(t)\} \in \Re \times \Re^3$ that is constructed to satisfy $\boldsymbol{q}_d^T\boldsymbol{q}_d + q_{0d}^2 = 1$. The corresponding rotation matrix, denoted by $R_d(\boldsymbol{q}_d, q_{0d}) \in \Re^{3\times3}$, that brings $\mathcal{I}$ onto $\mathcal{F}_d$ is then defined as

$$R_d = \left(q_{0d}^2 - \boldsymbol{q}_d^T\boldsymbol{q}_d\right)I_3 + 2\boldsymbol{q}_d\boldsymbol{q}_d^T - 2q_{0d}\boldsymbol{q}_d^{\times} \tag{5}$$

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